Preface

In this volume we collect papers which colleagues and friends of Decio Levi dedicated to his memory. The papers here presented reflect the scientific interests of Decio and most of them rely on his work, either directly by citing his own results, or indirectly by referring to his kind advices and suggestions. We thank all the authors that contributed to this astonishing memorial to Professor Levi. Moreover, we thank the *Open Communication in Nonlinear Mathematical Physics*, especially in the person of its Editor-in-Chief Prof. Norbert Euler for agreeing to host this special issue, the first ever published in that journal.

We present here a brief summary of the volume. After this preface, there is a short recollection of Decio Levi written by Orlando Ragnisco. After that, following the rules of *Open Communications in Nonlinear Mathematical Physics* the papers in this special issue are presented in order of acceptance, with the notable exception of the first paper, which is a posthumous paper by Decio Levi. Such a paper is a work of R. Hernández Heredero, D. Levi and C. Scimiterna in which they apply the multiple scale expansion up to the order ε^6 to a dispersive multilinear class of equations on a square lattice by obtaining four nonlinear equations, three of which are new. These equations are candidate to be be integrable integrable, and in fact their integrability has been discussed already in the literature.

The second paper, by F. Briscese and F. Calogero, is about certain numerical observations generalizing the Collatz conjecture and some of its known variants, like the Kakutani $5n+1$ or $7n+1$ conjectures. Analogous conjectures are presented and tested in this work.

In the third paper, written by E. V. Kaptsov and V. A. Dorodnitsyn, the authors construct and implement a new finite-difference schemes for the one-dimensional shallow water equations in the presence of a magnetic field in Lagrangian and mass Lagrangian coordinates. This approach is known as *symmetry invariant discretisation*, which is part of the broader *Geometric Integration Theory*, and Decio Levi and his collaborators gave many fundamental contributions to this field.

The fourth paper, by P. Albares, P. G. Estévez, A. González-Parra, and P. del Olmo, investigates the complex version of the modified Korteweg-de Vries equation by using two different techniques: the singular manifold method firstly and then the identification of the classical Lie symmetries of the system together with its spectral problem. Also in this case, note that Decio Levi gave many many contribution in the theory of Lie symmetries applied to nonlinear integrable equations.

In the next paper, A. M. Grundland focuses on the investigation and construction of multiple Riemann wave solutions obtained via the conditional symmetry method and

compares these results with the ones obtained through the generalized method of characteristics. Throughout his career Decio Levi also contributed to the theory and applications of conditional symmetries, and especially in the last years of his life to discretisation schemes based on the preservation of this kind of symmetries.

The next paper is by Yu. B. Suris: he studies a family of unconventional integrators for ODEs based on the polarization of vector fields and proposes a new elegant algebraic approach to derivation of the integrals of motion for polarization discretisations.

The seventh paper is by A. Doliwa: he presents an interpretation of suitable known results in the theory of discrete asymptotic and discrete conjugate nets through the discretisation techniques obtained by the Bäcklund transformations method. This is connected both to the activity of Decio Levi, and his longtime collaborator Antoni Sym, who was the Ph.D. advisor of A. Doliwa.

The next contribution is by E. Peroni and J. P. Wang: they construct the bi-Hamiltonian structure from the Nijenhuis recursion operator for a semi-discrete equation introduced by Decio Levi and his Russian collaborators, R. N. Garifullin and R. I. Yamilov.

Then, V. E. Adler in his contribution considers the Volterra lattice, showing that it admits a family of finite-dimensional non-autonomous reductions defined by a system of $m + 1$ equations of the Painlevé type.

In the next paper V. M. Buchstaber and A. V. Mikhailov analyse the problem of quantisation of stationary flows of the KdV hierarchy and introduce the notion of Frobenius-Hochschild algebras.

The eleventh paper is by X. Hu, G. Yu, and Y. Zhang: they present a discretisation of AKNS and KdV recursion operators by utilizing the unified bilinear forms and Hirota's derivative.

In his contribution, C-M. Viallet considers the algebraic entropy of two non-integrable maps and confronts three different methods to compute the algebraic entropy.

J. Hietarinta, in his work, investigates the degree growth of different partial difference equations defined on a 3×3 stencil, mostly connected to the discretisation of the Boussinesq equation.

The next contribution is by M. A. Rodríguez and P. Tempesta. They present a novel discretization procedure for the Euler ordinary differential equation based on the theory of Galois differential algebras and umbral calculus. Note that, again Decio Levi was one of the pioneers of the use of umbral calculus to discretise linear ordinary differential equations.

In the fifteenth paper S. Carillo and C. Schiebold overview nonlinear evolution equations of third order, their solutions and invariances using Bäcklund transformations.

In the next paper F. W. Nijhoff and D-J. Zhang construct the Lagrangian multiform structure of the lattice Boussinesq equation from the Lagrangian multiform structure of the lattice Gel'fand–Dikii hierarchy via properly chosen linear combinations.

The paper by G. Gubbiotti deals with the calculation of algebraic entropy for systems of quad equations: many examples from the literature are discussed within this approach. The original ideas of this paper where discussed by the author with Decio Levi when he was finishing his Ph.D. with him.

Then, A. N. W. Hone, J. A. G. Roberts, P. Vanhaecke and F. Zullo analyse four-dimensional birational maps of recurrence type and show how they provide genus two solutions of Volterra and modified Volterra integrable maps.

Finally, in the last paper G. Gubbiotti, D. I. McLaren and G. R. W. Quispel explain

how to obtain the Kahan-Hirota-Kimura discretization of a general two-dimensional cubic Hamiltonian and present different examples of their construction.

The volume is then closed with a scientific and personal biography of Decio Levi written by one of us.

With gratitude for the gift of friendship,
Giorgio Gubbiotti
Orlando Ragnisco
Paolo Maria Santini
Federico Zullo